ALGEBRA

List 5.

Analytic geometry in $\Re^2$ and $\Re^3$.

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1. A line ℓ and a point M on the plane are given. Write the parametric and the normal forms of equation for a line which contains M and is

(1) parallel to ℓ; (2) orthogonal to ℓ

in the following cases:

a) $M(2, -3)$, $\ell : 2x - 3y + 5 = 0$;

b) $M(1, -2)$, $\ell : 5x - y + 3 = 0$;

c) $M(4, -1)$, $\ell : -3x + y + 2 = 0$.

2. For the triangle ABC with $A(-2, 3)$, $B(4, 1)$, $C(6, -5)$, write the the parametric and the normal forms of equation for a line which contains

a) the median containing the vertex A;

b) the bisector containing the vertex A;

c) the altitude containing the vertex A.

3. The middle points of the sides of a triangle are $M_1(2, 3)$, $M_2(-1, 2)$ i $M_3(4, 5)$. Find equations of the sides of the triangle.

4. Write an equation of the line such that the point $P(2, 3)$ is the orthogonal projection of the origin on this line.

5. The lengths of vectors $\vec{v}$ and $\vec{w}$ are equal to 2 and 3, respectively. Knowing that $(\vec{v}, \vec{w}) = -1$, calculate

a) $(\vec{v} + 2\vec{w}, 2\vec{v} - \vec{w})$;

b) the angle between $\vec{v} + \vec{w}$ and $\vec{v} - \vec{w}$.

6. Find the values of the parameters t, s for which the vectors $\vec{v} = (2-2t, 2, -4)$ and $\vec{w} = (1, 3-s, 1)$ are parallel.

7. Find the values of the parameter t for which vectors $\vec{v} = (2 - 2t, 2, -4)$ and $\vec{w} = (1, 3 - t, 1)$ are orthogonal.

8. Compute the area of the parallelogram spanned by vectors $\vec{v} = (2, 2, -1)$ and $\vec{w} = (1, 3, 2)$.

9. Compute the area of the triangle with vertices $A = (1, 0, 1), B = (2, 0, 4)$ and $C = (0, 1, 1)$.

10. For the triangle from the previous problem calculate all the altitudes.

11. Compute the volume of the parallelepiped spanned by vectors $\vec{u} = (2, 2, -4)$ $\vec{v} = (1, 2, 0)$ and $\vec{w} = (1, 3, 1)$.

12. Compute the volume of the tetrahedron with vertices $A = (0, 1, 0), B = (1, 1, 2), C = (0, 2, 1)$ and $D = (3, 2, -1)$.

13. For the tetrahedron from the previous problem compute the altitude through the vertex D.

14. Find normal and parametric equations of the plane

(a) through the points $P=(1,2,1), Q=(2,1,5)$ and $C=(3,0,1)$;

(b) through the point $P=(-2,3,2)$ and including the Ox axis;

(c) through the point $P=(1,0,1)$ and orthogonal to the Oy axis.

15. Explain why the parametric equations

$$\begin{cases} x=2+t \\ y=1+t \\ z=-1+3t \end{cases} \quad \text{and} \quad \begin{cases} x=2t \\ y=-1+2t \\ z=-7+6t \end{cases}$$

describe the same line.

16. Do the parameteric equations

$$\begin{cases} x=2+3t+s \\ y=1+t+2s \\ z=-1+t-s \end{cases} \quad \text{and} \quad \begin{cases} x=5+4t+2s \\ y=2+3t+4s \\ z=-2s \end{cases}$$

describe the same plane? Justify your answer.

17. Find a parametric equation of the plane given by the equation $x+2y-z+5=0$.

18. Find a normal equation of the plane given by the parametric equation

$$\begin{cases} x=2+t+2s \\ y=1+2t+s \\ z=3+t-s \end{cases}$$

19. Find a parametric equation of the line in which two planes

$$\begin{cases} x+2y+z+3=0 \\ 2x-y+z+5=0 \end{cases}$$

intersect each other.

20. Find the intersection point of the line $l: x=t, y=1+2t, z=3+t$ and the plane $\pi: x+2y-z-3=0$.

21. For the point $P=(1,0,1)$ and the plane $\pi: x+2y-z+3=0$, find

(a) the projection of P on π;

(b) the distance from P to π;

(c) the point, symmetric to P with respect to π.

22. For the point $P=(1,2,3)$ and the line $l: x=2t, y=1-t, z=-2+3t$, find

(a) the projection of P on l;

(b) the distance from P to l;

(c) the point, symmetric to P with respect to l.

23. Find the distance between two parallel lines

$$\begin{cases} x+y+z+2=0 \\ 2x-y+z+5=0 \end{cases} \quad \text{and} \quad \begin{cases} x+y+z+2=0 \\ 2x-y+z+7=0 \end{cases}$$

24. A line ℓ and a point P on the plane are given. Find the point Q, which is the projection of P on ℓ, and the point R, which is symmetric to P w.r.t. ℓ

a) $P(-6,4)$, $\ell : 4x - 5y + 3 = 0$;

b) $P(-5,13)$, $\ell : 2x - 3y - 3 = 0$;

c) $P(-8,12)$, ℓ contains $M_1(2,-3), M_2(-5,1)$;

d) $P(8,-9)$, ℓ contains $M_1(3,-4), M_2(-1,-2)$.

25. Check if the lines ℓ_1 and ℓ_2 are parallel. For parallel lines find the distance between them. For non-parallel lines find the acute angle between them.

a) $\ell_1 : x + y + 3 = 0, \ell_2 : \begin{cases} x = 1 - t, \\ y = 2 + t, \end{cases}$;

b) $\ell_1 : 2x - y + 1 = 0, \ell_2 : \begin{cases} x = 1 + t, \\ y = 2 - t, \end{cases}$.